Adaptive Discontinuous Galerkin Methods in Multiwavelets Bases

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In this research we demonstrate the advantages of combining the structure of multiwavelets with the Discontinuous Galerkin (DG) method for overcoming the shortcomings in the standard DG methods. To illustrate the important gains of using the Multiwavelet DG method we apply it to non-linear conservation equations. The significant benefits of merging DG methods with multiwavelets are three-fold. First, the DG method inherits a hierarchical structure from multiwavelets that produces a weak decoupling across different length scales. Second, the problems of when and how to perform hp-adaptivity in the DG method is answered by the scaling properties of multiwavelets. Third, the matrix of the muliwavelet DG operator and its inverse share the same sparse pattern, that provides nearly linear scaling of memory and computational performance with increasing degrees of freedom and dimensionality. In addition, the highly desired sparsity pattern combined with multiresolution provides a direct way for developing fast numerical solvers. These properties are especially important for higher dimensional problems with large degrees of freedom.

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